Heat Transfer in Forced Convection with Internal Heat Generation

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A solution to the problem of heat transfer with simultaneous heat generation in viscous tubular flow is presented. The temperature profiles and heat transfer coefficients which are obtained apply to compressible as well as incompressible Newtonian and power-law non-Newtonian fluids with constant physical properties and to systems in which the heat generation is an arbitrary function of radius. An example of heat transfer with frictional heat generation in a non-Newtonian fluid is also presented, and the solution to the problem in which a fluid enters a tube in laminar flow with an arbitrary temperature profile is given, with a consideration of a first approximation to the case of heat transfer in a turbulent fluid in which heat is being generated.

Heat may be generated or absorbed in a fluid by various mechanisms. In cases such as a chemical reaction or fluid flow under a large pressure gradient the heat generation or absorption is incidental to the main operation, and it would be desirable to be able to determine the effect of the generation on the system. In the flow of very viscous liquids, such as high polymers, the frictional heat generation may be quite large, and one would like to know the temperature history and mean exit temperature of the material when it is forced through a hot or cold conduit.

Although a number of viscous-flow heat-generation problems have been analyzed, the general solution to the Graetz problem with simultaneous heat generation is not available. Gee and Lyon (*5*) have worked out one particular solution on a digital computer; Topper (*14*) has solved the problem with a generation term which is constant across the tube; and Brinkman (*3*), Bird (*1*), and others (*13*) have considered the special case where the inlet temperature equals the wall temperature and heat is generated by friction. A number of solutions to related problems are also available (*4, 6, 7, 12*).

In this paper the general solution to the Graetz problem with heat generation will be obtained for Newtonian and certain types of non-Newtonian fluids by an extension of the methods used by Brinkman and Bird. The solution will not exclude compressibility effects, but, as an example, heat transfer coefficients and temperature profiles will be calculated for an incompressible non-Newtonian fluid in which heat is generated by friction. The solution to the problem in which a fluid in laminar flow enters a tube with an arbitrary temperature profile will also be obtained as a by-product. Since physical properties will be assumed constant and the velocity profile will be taken as fully developed, the solutions must be considered as approximations to the real case in the same sense as the Graetz equation is an approximation to the case of no heat generation (*10*).

In turbulent flow, in addition to the well-known example of high-velocity gas flow (*10*), the case of constant heat generation with a constant wall heat flux in a long tube has been investigated (*11*), and a first approximation to the problem in which the wall temperature is specified will be considered here.

TURBULENT FLOW

In a steady, well-developed turbulent flow entering a uniform channel at a temperature T_0, where the wall temperature is constant at T_w, the local rate of heat transfer to the fluid in the absence of heat generation is given by

$$dq = h(T_w - T_m)P\,dx \qquad (1)$$

If the equation is interpreted in terms of the idealized film theory, all resistance to transfer lies in a thin laminar film at the wall, and the central core of fluid is completely mixed. With this model, then, there would be negligible interaction between the heat transfer coefficient and the heat generation, since practically all the heat is generated in the turbulent core where mixing is immediate. In addition, the particular manner in which the heat generation varies with radial position is immaterial as long as a significant fraction of the generation does not take place in the film. Thus the following heat balance can be written in the presence of heat generation,

$$wc_p\,dT_m = h(T_w - T_m)P\,dx + WA_x\,dx \qquad (2)$$

and if the film theory is a suitable approximation, h has the same value it would have if there were no heat generation. Rearranging (2) gives

$$\frac{dT_m}{dx} = \frac{hP}{wc_p}\left[\left(T_w + \frac{WA_x}{hP}\right) - T_m\right] \qquad (3)$$

If the group in the inner parentheses is considered to be T_{we}, and since the inlet temperature is T_0, both with and without heat generation, it is clear that heat generation causes the system to act as if the wall temperature were higher than the actual value by the amount WA_x/hP, which is $WR/2h$ for a tube. This result differs from high-velocity flow, as that particular case has been excluded from the analysis on the assumption that the heat generation is not concentrated near the wall. The solution to Equation (3) for a conduit of constant area and shape with W, h, and Cp constant, is

$$\frac{T_{we} - T_m}{T_{we} - T_0} = e^{-hPx/wc_p} \qquad (4)$$

and Equation (4) shows that the effective wall temperature, T_{we}, is the temperature attained by the fluid in flowing through a long conduit. If Equation (4) is put in the usual design form, then

$$q = wc_p(T_m - T_0) = hA_s(T_{we} - T_m)_{lm} \qquad (5)$$

and q is not the total thermal energy transferred across A_s but, as defined by Equation (5), is the net thermal energy supplied to the fluid by convection and generation.

If WA_x/hP varies with x or T_m, the problem is equivalent to one with no heat generation and a varying wall temperature. If W varies linearly with T_m, Equation (5) still holds, and the mean driving force is the log mean of $(T_{we} - T_m)$ at both ends of the conduit.

The foregoing analysis can be considered only a first approximation, and, when resistance to transfer in the turbulent core is a significant fraction of the total resistance, the heat transfer coefficient will be altered by the heat generation. The analysis would be expected to increase in accuracy as the Prandtl number and the ratio of the heat transfer through the wall to the heat generation

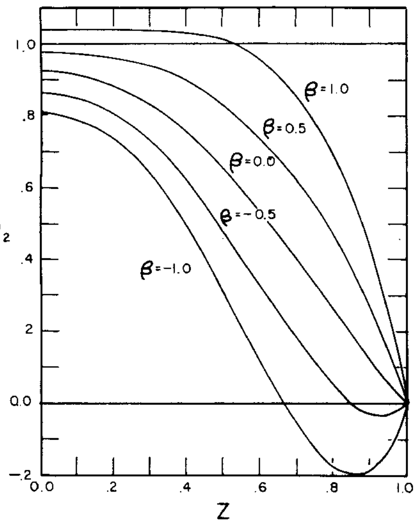

Fig. 1. Temperature profiles at $N_{Gz} = 31.4$; frictional heat generation with $n = 3$.

increase. The interaction between the coefficient and generation, for a constant flux in a long tube, can be obtained by analogy from available calculations (*11*) and might be used to estimate h in the problem considered here. When the interaction between the coefficient and generation is a predominate factor, the situation is of the same type as encountered in laminar flow.

LAMINAR FLOW

The energy equation for laminar flow is (*12*)

$$\rho c_p \frac{dT}{d\theta} = k\nabla^2 T + \frac{\epsilon'}{J}\frac{dp}{d\theta} + \Phi \qquad (6)$$

The last term on the right is the volumetric rate of heat generation, and the adjacent term is minus the rate at which heat is removed by fluid expansion.

Considered here is a fluid at a constant temperature T_0 entering a tube with a fully developed laminar velocity profile which is constant along the tube; that is, u is a function of r only and there are no radial velocity components. The wall is at a constant temperature T_w, and heat is generated (or consumed) at a rate depending only upon radial position. The physical properties are independent of temperature, and the flow has existed long enough for the temperature at all points to be independent of time. Equation (6) then reduces to

$$\rho c_p u \frac{\partial T}{\partial x} = k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\epsilon' u}{J}\frac{\partial p}{\partial x} + \Phi(r) \qquad (7)$$

when axial conduction is neglected. To include non-Newtonian as well as Newtonian fluids the stress-rate-of-strain curve is assumed to be given by a power law,

$$\frac{\partial u}{\partial r} = -A\tau^{n-1} \qquad (8)$$

For $n = 2$ this is the equation of a Newtonian fluid, and for $n > 2$ the equation approximately describes the rheological properties of many high polymers. The velocity profile then becomes (*12*)

$$\frac{u}{u_m} = \frac{n+2}{n}\left[1 - \left(\frac{r}{R}\right)^n\right] \qquad (9)$$

$$u_m = \frac{AR^n}{n+2}\left(-\frac{dp}{2dx}\right)^{n-1} \qquad (10)$$

and Equation (7) can be written as

$$\rho c_p u_m \frac{n+2}{n}\left[1 - \left(\frac{r}{R}\right)^n\right]\frac{\partial T}{\partial x} = k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + Wf(r) \qquad (11)$$

where W is defined by

$$W = \frac{2}{R^2}\int_0^R r\left[\frac{\epsilon' u}{J}\frac{\partial p}{\partial x} + \Phi\right] dr = \frac{\epsilon' u_m}{J}\frac{\partial p}{\partial x} + \frac{2}{R^2}\int_0^R r\Phi\, dr \qquad (12)$$

and $f(r)$ describes the manner in which the net generation varies wih radius.

The following transformations:

$$\Theta = \frac{T - T_w}{\frac{1}{a}\frac{R^2 W}{k}} \qquad (13)$$

$$Z = \frac{r}{R} \qquad (14)$$

$$X = \frac{n}{n+2}\frac{\alpha x}{R^2 u_m} = \frac{n\pi}{n+2}(N_{Gz})^{-1} \qquad (15)$$

make Equation (11) dimensionless,

$$(1 - Z^n)\frac{\partial\Theta}{\partial X} = \frac{1}{Z}\frac{\partial}{\partial Z}\left(Z\frac{\partial\Theta}{\partial Z}\right) + af(Z) \qquad (16)$$

The dimensionless number a is included merely for convenience.

The boundary conditions of the problem are

$$\Theta(0, Z) = \Theta_0 = \frac{T_0 - T_w}{\frac{1}{a}\frac{R^2 W}{k}} \qquad (17a)$$

$$\Theta(X, 1) = 0 \qquad (17b)$$

$$\frac{\partial\Theta(X, 0)}{\partial Z} = 0 \qquad (17c)$$

For large X the temperature profile is independent of X, and Equation (16) reduces to

$$\frac{1}{Z}\frac{d}{dZ}\left(Z\frac{d\Theta_s}{dZ}\right) + af(Z) = 0 \qquad (18)$$

Conditions (17*b*) and (17*c*) are unchanged, and Equation (18) yields the temperature profile for large X,

$$\Theta_s = a\int_Z^1 \frac{1}{Z'}\int_0^{Z'} Zf(Z)\, dZ\, dZ' \qquad (19)$$

Following previous procedures (*3, 1, 13*) the solution to Equation (16) is assumed to be

$$\Theta = \Theta_s + \Theta_1 \qquad (20)$$

and by substituting this into Equation (16) one removes the generation term,

$$(1 - Z^n)\frac{\partial\Theta_1}{\partial X} = \frac{1}{Z}\frac{\partial}{\partial Z}\left(Z\frac{\partial\Theta_1}{\partial Z}\right) \qquad (21)$$

The boundary conditions now are

$$\Theta_1(0, Z) = \Theta_0 - \Theta_s \qquad (22a)$$

$$\Theta_1(X, 1) = 0 \qquad (22b)$$

$$\frac{\partial\Theta_1(X, 0)}{\partial Z} = 0 \qquad (22c)$$

Except for condition (22*a*) the problem is identical to the Graetz problem (*10*) when $n = 2$ and to the non-Newtonian generalization treated by Bird (*2*) for other values of n. It has been shown (*2*) that the solution to Equation (21) is

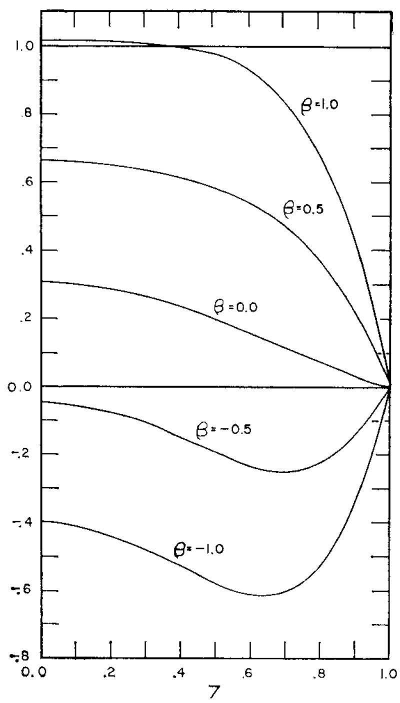

Fig. 2. Temperature profiles at $N_{Gz} = 7.85$; frictional heat generation with $n = 3$.

$$\Theta_1 = -\sum_{i=1}^{\infty} B_i\varphi_i(Z)e^{-a_iX} \quad (23)$$

and the eigenfunctions $\varphi_i(Z)$ and the eigenvalues a_i are tabulated for various n (2).

From Equation (22a) and (23) at $X = 0$,

$$\Theta_0 - \Theta_s = -\sum_{i=1}^{\infty} B_i\varphi_i(Z) \quad (24)$$

and since the $\varphi_i(Z)$ satisfy the orthogonality condition

$$\int_0^1 (1 - Z^n)Z\varphi_i(Z)\varphi_j(Z)\, dZ = 0 \quad i \neq j \quad (25)$$

multiplication of Equation (24) by $\varphi_j(Z)Z(1 - Z^n)\, dZ$ and integration allows determination of the B_i,

$$B_i = B_i^* - \Theta_0 B_i' \quad (26)$$

where

$$B_i^* = \frac{\int_0^1 \Theta_s(1 - Z^n)Z\varphi_i(Z)\, dZ}{\int_0^1 Z(1 - Z^n)\varphi_i^2(Z)\, dZ} \quad (27)$$

$$B_i' = \frac{\int_0^1 (1 - Z^n)Z\varphi_i(Z)\, dZ}{\int_0^1 Z(1 - Z^n)\varphi_i^2(Z)\, dZ} \quad (28)$$

From Equations (20), (23), and (26) the solution to the original problem is

$$\Theta = \Theta_s(Z) - \sum_{i=1}^{\infty} B_i^*\varphi_i(Z)e^{-a_iX} + \Theta_0 \sum_{i=1}^{\infty} B_i'\varphi_i(Z)e^{-a_iX} \quad (29)$$

Dividing Equation (29) by θ_0 gives a more convenient form:

$$\Theta_2 = \beta\left[\Theta_s(Z) - \sum_{i=1}^{\infty} B_i^*\varphi_i(Z)e^{-a_iX}\right] + \sum_{i=1}^{\infty} B_i'\varphi_i(Z)e^{-a_iX} \quad (30)$$

where

$$\Theta_2 = \frac{T - T_w}{T_0 - T_w} \quad (31)$$

$$\beta = \frac{1}{\Theta_0} \quad (32)$$

If $1/a$ is defined to be the value of the double integral in Equation (19) at some reference point, say at the center line,

$$\frac{1}{a} = \int_0^1 \frac{1}{Z'} \int_0^{Z'} Zf(Z)\, dZ\, dZ' = \frac{T_{sc} - T_w}{R^2W/k} \quad (33)$$

then θ_s is 1.0 at $Z = 0$; the denominator of Equation (13) becomes $T_{sc} - T_w$, and from Equation (32),

$$\beta = \frac{T_{sc} - T_w}{T_0 - T_w} \quad (34)$$

where T_{sc} is the center-line temperature at $X = \infty$, given by Equation (33).

The term in brackets in Equation (30) now rises from 0 at $X = 0$ to a maximum value of 1.0 in the tube center at $X = \infty$, while the term on the right decreases from 1.0 at $X = 0$ to 0 at $X = \infty$. β may take on values ranging from $-\infty$ to $+\infty$, and, when the absolute value of β is small, heat generation may be neglected.

β also has another interpretation. With the earlier assumptions, an energy balance over an adiabatic conduit yields

$$wc_p(T_{ma} - T_0) = W\pi R^2 x \quad (35)$$

combining this with Equation (32) gives

$$\beta = \frac{1}{a}\frac{N_{GZ}}{\pi}\beta' \quad (36)$$

where β' is the ratio of the adiabatic temperature rise to the inlet temperature difference.

When there is no generation or expansion, $\beta = 0$, and Equation (30) reduces to

$$\Theta_2' = \sum_{i=1}^{\infty} B_i'\varphi_i(Z)e^{-a_iX} \quad (37)$$

This is the solution to the generalized Graetz problem for which the θ_2' are tabulated (2). [For a Newtonian fluid Equation (37) is Graetz's solution.]

When $T_0 = T_w$, Equation (29) yields

$$\Theta_2^* = \Theta_s - \sum_{i=1}^{\infty} B_i^*\varphi_i(Z)e^{-a_iX} \quad (38)$$

the solution to the problem in which the inlet temperature equals the wall temperature.

Thus from Equations (30), (33), (37), and (38),

$$\Theta_2 = \beta\Theta_2^* + \Theta_2' \quad (39)$$

or

$$\frac{T - T_w}{T_0 - T_w} = \frac{T^* - T_w}{T_0 - T_w} + \frac{T' - T_w}{T_0 - T_w} \quad (40)$$

and so the reduced temperature at any point in a tube is equal to, in the general case, the reduced temperature which would be obtained with heat generation if the inlet temperature were equal to the wall temperature, plus the reduced temperature which would be obtained in the absence of heat generation.

Since the a_i, $\varphi_i(Z)$, and B_i' are known for various n (2), specifying $f(Z)$ fixes θ_s by Equation (19), and Equation (27) allows the B_i^* to be determined. Numerical values may then be obtained from Equation (30). (It should be noted that the n used in this paper is $[1 + n]$ in Bird's notation [1, 2]).

MEAN TEMPERATURE CHANGE AND HEAT TRANSFER COEFFICIENT

The mean temperature across a tube is defined by

$$\Theta_{2m} = 2\int_0^1 Z\frac{u}{u_m}\Theta_2\, dZ \quad (41)$$

and the average of Equation (39) shows that the mean reduced temperatures are additive in the same sense as the reduced point temperatures. It follows that the total thermal energy change of the fluid is the sum of the change due to normal conduction without generation and the change due to the net effect of heat generation in a tube where the wall temperature equals the inlet temperature. The former term is available, and the latter can be calculated once $f(Z)$ is specified.

Alternately an arithmetic mean heat transfer coefficient can be defined to give the effects of conduction and heat generation by

$$q = wc_p(T_m - T_0) = 2\pi Rh_a x\left[\frac{(T_w - T_0) + (T_w - T_m)}{2}\right] \quad (42)$$

This leads to the usual equation for h_a (10),

$$\frac{h_aD}{k} = \frac{2}{\pi}\left(\frac{wc_p}{kx}\right)\frac{1 - \Theta_{2m}}{1 + \Theta_{2m}} \quad (43)$$

and from Equations (39) and (41) and the definitions of Nusselt and Graetz numbers,

$$N_{Nu} = \frac{2}{\pi}N_{GZ}\left[\frac{1 - \Theta_{2m}' - \beta\Theta_{2m}}{1 + \Theta_{2m}' + \beta\Theta_{2m}}\right] \quad (44)$$

When $T_w > T_0$, β is negative and N_{Nu} is increased by generation and when $T_w < T_0$, β is positive and N_{Nu} is decreased by generation.

FRICTIONAL HEAT GENERATION—LAMINAR FLOW

An example of the application of the foregoing equations is frictional heat generation in an incompressible fluid. Here

$$W = -\frac{u_m}{J}\frac{dp}{dx} \quad (45)$$

and

$$f(Z) = \frac{n + 2}{2}Z^n \quad (46)$$

Equation (33) gives

$$a = 2(n + 2) \quad (47)$$

and the profile for large (X) is given by Equations (19) and (46),

$$\Theta_s = 1 - Z^{n+2} \quad (48)$$

The equations for $f(Z)$ and θ_s are available if the fluid is compressible (*12*). From Equations (45), (32), and (36),

$$\beta' = \frac{\Delta p/\rho c_p}{J(T_0 - T_w)} \tag{49}$$

and $\Delta p/\rho c_p$ is the mean adiabatic temperature rise of an incompressible fluid, $(T_{ma} - T_0)$.

With θ_s known, the $B_i{}^*$ may be obtained from Equation (27). Since these calculations have already been carried out in studies of frictional heat generation (*1, 13*) in which the inlet temperature equals the wall temperature, the temperature profiles shown in Figures 1 and 2 were obtained by calculating $\theta_2{}^*$ from Equation (38) and adding these to the tabulated (*2*) values of $\theta_2{}'$.

$\theta_{2m}{}^*$ was obtained by numerically integrating Equation (41) by use of the preceding values of $\theta_2{}^*$, and, with Bird's (*2*) values of $\theta_{2m}{}'$, θ_{2m} was obtained from Equations (39), and (41). All these calculations apply only for $n = 3$.

The series for $\theta_{2m}{}'$ converges slowly at high Graetz numbers, but the asymptotic form has been determined by Lévêque (*8*) for Newtonian fluids and can be obtained from Marshall and Pigford (*9*) for non-Newtonian fluids. The value of $\theta_{2m}{}^*$ at high Graetz numbers is known only for the adiabatic case, but as the series in Equation (38) converges more rapidly than the one for $\theta_{2m}{}'$, θ_{2m} could be extended to Graetz numbers of about 200 without too great an error.

Figure 3 shows $1 - \theta_{2m}$, which is the reduced temperature rise of the fluid, as a function of Graetz number for various values of β and β'. When the inlet fluid is warmer than the tube wall, β is positive, and the temperature change of the fluid is decreased as β increases, since the heat generation partly compensates for the heat loss through the tube wall. When β is negative, the conduction and generation are both in the same direction, and the temperature change is increased as the magnitude of β increases.

At low values of the Graetz number the temperature attains its steady value, which from Equations (30), (41), (9), and (48) is found to be

$$\frac{T_{ms} - T_0}{T_w - T_0} = 1 - \frac{n+3}{n+4}\beta \tag{50}$$

This is $1 - 6/7\beta$ for $n = 3$, and it is clear that when β is negative the mean temperature rises above the wall temperature when the Graetz number is small. From Figure 3, the point at which the mean temperature reaches the wall temperature depends upon β and N_{GZ}. For $\beta = -1$ the mean temperature reaches the wall temperature at a Graetz number of 20 and exceeds it for lower Graetz numbers.

When β is positive, Equation (50) shows that the mean temperature may fall below the wall temperature for large enough values of β.

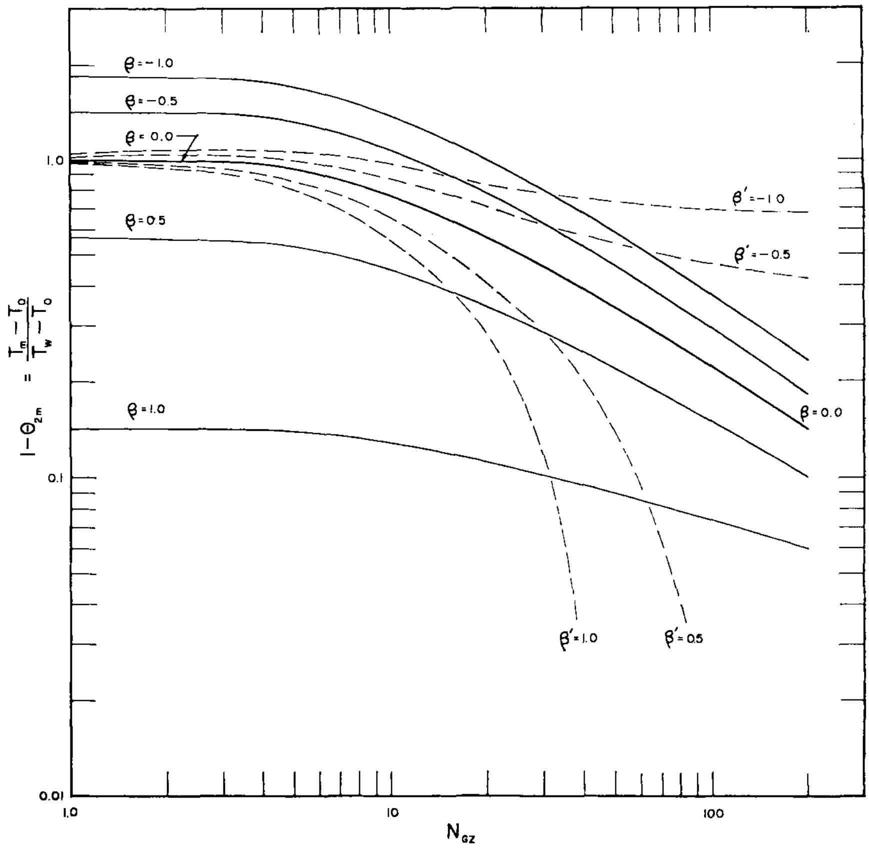

Fig. 3. Mean temperature rise with frictional heat generation; $n = 3$.

The lines of constant β' of Figure 3 apply to operation at a constant ratio of the adiabatic temperature rise to the inlet temperature difference. They would correspond to operation with one fluid at at a constant pressure drop with a varying tube length. Although the heat generated per unit mass of fluid is constant along a line of constant β', all the lines converge to the $\beta = 0$ line at low Graetz numbers and diverge increasingly from this line as β' and N_{GZ} increase.

The results in Figure 3 are shown in Figure 4 in terms of the arithmetic mean heat transfer coefficient, defined by Equation (44). The asymptote at small Graetz numbers, when the steady temperature is attained, is from Equations (44) and (50)

$$N_{Nu} = \frac{2}{\pi}\left[\frac{(n+4) - \beta(n+3)}{(n+4) + \beta(n+3)}\right] N_{GZ} \tag{51}$$

HEAT TRANSFER WITH INLET TEMPERATURE A FUNCTION OF RADIUS: LAMINAR FLOW

The solution can be obtained from the earlier results, if the generation and expansion terms are dropped from Equation (7), the reduced coordinates (14) and (15) are used, and (13) is replaced by

$$\Theta_3 = \frac{T - T_w}{T_{m0} - T_w} \tag{52}$$

where T_{m0} is the mean inlet temperature. The differential equation is

$$(1 - Z^n)\frac{\partial \Theta_3}{\partial X} = \frac{1}{Z}\frac{\partial}{\partial Z}\left(Z\frac{\partial \Theta_3}{\partial Z}\right) \tag{53}$$

the boundary conditions are

$$\Theta_3(0, Z) = I(Z) \tag{54a}$$

$$\Theta_3(X, 1) = 0 \tag{54b}$$

$$\frac{\partial \Theta_3(X, 0)}{\partial Z} = 0 \tag{54c}$$

and $I(Z)$ is the reduced inlet temperature.

This problem is formally identical to the one defined by Equations (21) and (22) if $I(Z) = \theta_0 - \theta_s$. From Equations (23) to (28) the solution to the present problem is

$$\Theta_3 = \sum_{i=1}^{\infty} e^{-\alpha_i X}\varphi_i(Z) \cdot \frac{\int_0^1 I(Z)Z(1 - Z^n)\varphi_i(Z)\,dZ}{\int_0^1 Z(1 - Z^n)\varphi_i{}^2(Z)\,dZ} \tag{55}$$

The generalization to the case in which heat is being generated follows readily.

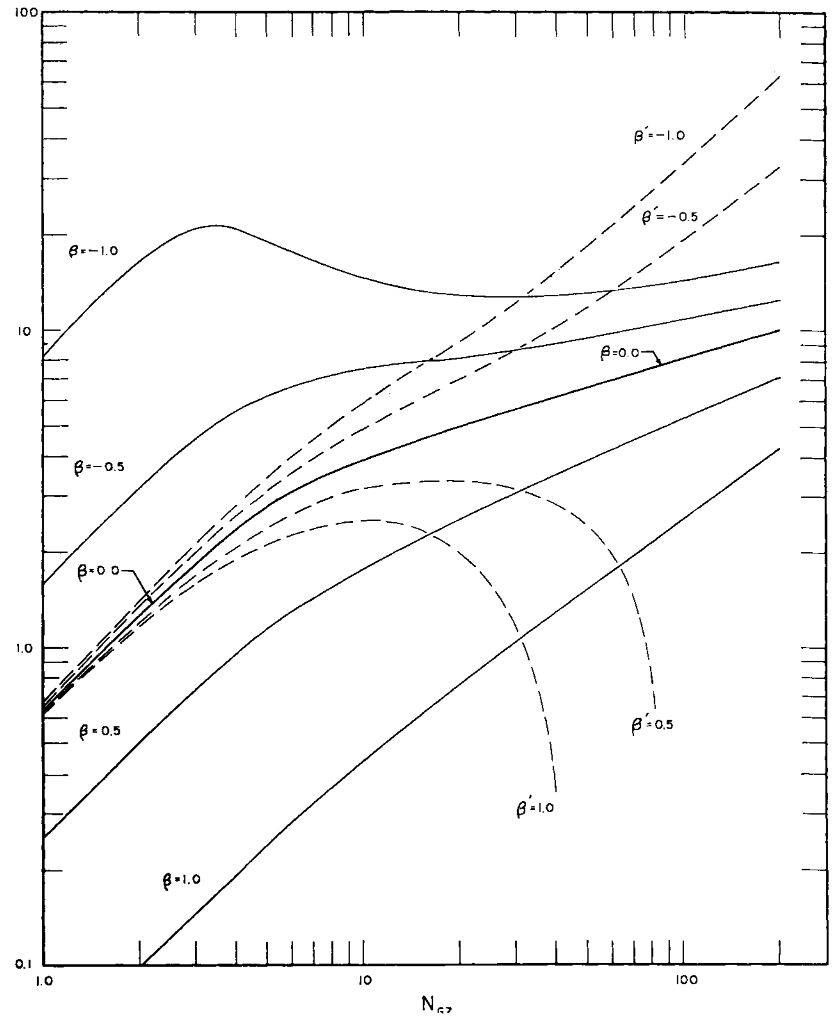

Fig. 4. Nusselt number with frictional heat generation; $n = 3$.

Greek Letters

α = thermal diffusivity, sq. ft./hr.
β = dimensionless parameter, $1/\theta_0$
β' = dimensionless parameter, $(T_{ma} - T_0)/(T_0 - T_w)$
ϵ' = mean value of absolute temperature times coefficient of thermal expansion
θ = time, hr.
Θ = reduced temperature = $(T - T_w)/(1/a)(R^2W/k)$
Θ_1 = reduced temperature = $\Theta - \Theta_s$
Θ_2 = reduced temperature = $(T - T_w)/(T_0 - T_w)$
Θ_3 = reduced temperature = $(T - T_w)/(T_{m0} - T_w)$
Θ_2' = reduced temperature = $(T' - T_w)/(T_0 - T_w)$
Θ_2^* = reduced temperature = $(T^* - T_w)/(T_{sc} - T_w)$
ρ = density, lb.-mass/cu. ft.
τ = shear, stress, lb./force/sq. ft.
φ_i = eigenfunction
Φ = volumetric rate of heat generation, B.t.u./(hr.)(cu. ft.)

Subscripts

c = center line
i, j = indexes
m = mean
lm = logarithmic mean
0 = initial value, $X = 0$
s = steady value, $X = \infty$
w = wall temperature
∇^2 = Laplacian operator
$\frac{d}{d\theta}$ = derivative following the motion of the fluid

NOTATION

a = dimensionless constant, Equation (33)
a_i = eigenvalues
A = constant relating stress and rate of strain, 1/hr. × (lb. force/sq. ft.)$^{1-n}$
A_s = surface area for heat transfer, sq. ft.
A_x = cross-sectional area, sq. ft.
B_i = dimensionless coefficient, Equation (23)
B_i^* = dimensionless coefficient, Equation (27)
B_i' = dimensionless coefficient, Equation (28)
c_p = heat capacity at constant pressure, B.t.u./(lb.-mass)(°R.)
D = diameter, ft.
f = dimensionless function which describes variation of heat generation with position
h = local heat transfer coefficient, B.t.u./(hr.)(sq. ft./°F.)
h_a = arithmetic mean heat transfer coefficient, B.t.u./(hr.)(sq. ft./°F.)
I = reduced inlet temperature
J = conversion factor = 778 ft.-lb.-force/B.t.u.
k = thermal conductivity, B.t.u./(hr.)(ft./°F.)
n = dimensionless constant, Equation (8)
N_{Nu} = Nusselt number, h_aD/k
N_{Gz} = Graetz number, wc_p/kx
p = pressure, lb.-force/sq. ft.
Δp = pressure drop, lb.-force/sq. ft.
P = perimeter, ft.
q = net rate of heat input, B.t.u./hr.
r = radius vector, ft.
R = tube radius, ft.
T = temperature, °R.
T_{ma} = mean adiabatic temperature, °R.
T_{we} = effective wall temperature, °R.
T' = temperature in absence of heat generation, °R.
T^* = temperature if $T_0 = T_w$, °R.
u = velocity, ft./hr.
w = mass flow rate, lb.-mass/hr.
W = mean net volumetric rate of heat generation across tube B.t.u./(hr.)(cu. ft.)
x = axial distance, ft.
X = reduced axial distance = $[n/(n+2)]\,\alpha x/(R^2u_m)$
Z = reduced radial distance = r/R

LITERATURE CITED

1. Bird, R. B., *Soc. Plastics Engineers' J.*, 11, 35 (September, 1955).
2. ———, *Chem. Eng. Sci.*, **6**, 35 (1956).
3. Brinkman, H. C., *Appl. Sci. Res.*, **A2**, 120 (1950).
4. Chambré, P. L., *ibid.*, **A6**, 393 (1957).
5. Gee, R. E., and J. B. Lyon, *Ind. Eng. Chem.*, **49**, 956 (1957).
6. Hausenblas, H., *Ing.-Arch.*, **18**, 151 (1950).
7. Hersey, M. D., *Physics*, **7**, 403 (1936).
8. Lévêque, M. A., *Compt. rend.*, **185**, 1190 (1927).
9. Marshall, W. R., Jr. and R. L. Pigford, "The Application of Differential Equations to Chemical Engineering Problems," p. 140, Univ. Delaware, Newark (1947).
10. McAdams, W. H., "Heat Transmission," McGraw-Hill Book Company, Inc., New York (1954).
11. Poppendiek, H. F., *Chem. Eng. Progr. Symposium Ser. No.* 11, **50**, 93 (1954).
12. Toor, H. L., *Ind. Eng. Chem.*, **48**, 922 (1956).
13. ———, *Trans. Soc. of Rheology*, **1**, 177 (1957).
14. Topper, Leonard, *Chem. Eng. Sci.*, **5**, 13 (1956).

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